OCR
RECOGNISING ACHIEVEMENT

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MATHEMATICS 2637

Mechanics 1

Tuesday **10 JUNE 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

**TIME** 1 hour 20 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\text{m}\,\text{s}^{-2}$.
- You are permitted to use a graphic calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

**This question paper consists of 4 printed pages.**

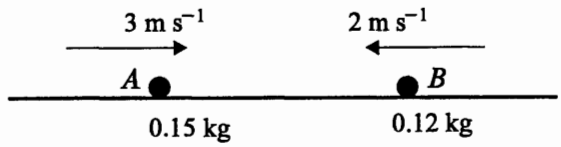

Registered Charity Number: 1066969
[Turn over

1 A particle travels in a straight line. Its displacement from the fixed point $O$ of the line, at time $t$ seconds after leaving $O$, is $5t^2 - \frac{1}{2}t^3$ metres.

**(i)** Write down expressions, in terms of $t$, for the velocity and acceleration of the particle. [3]

**(ii)** Find the velocity of the particle when its acceleration is $4\,\text{m s}^{-2}$. [3]

2




Two spheres, $A$ and $B$, of masses 0.15 kg and 0.12 kg respectively, are moving horizontally on a smooth plane. They move towards each other, in the same straight line, with speeds of $3\,\text{m s}^{-1}$ and $2\,\text{m s}^{-1}$ respectively (see diagram).

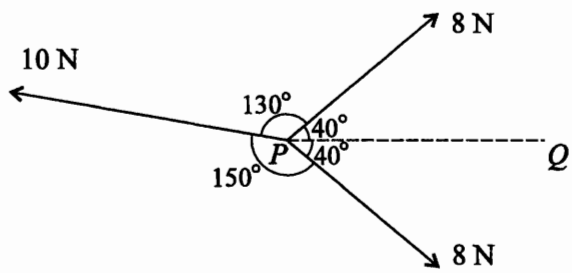
**(i)** Find the total momentum of the spheres in the direction of $A$'s motion. [2]

The spheres collide and the speed of $B$ immediately after the collision is $b\,\text{m s}^{-1}$.

**(ii)** Given that the speed of $A$ immediately after the collision is zero, and that $B$'s direction of motion is reversed in the collision, find the value of $b$. [2]

**(iii)** Given instead that the directions of motion of both spheres are reversed in the collision, give a reason why the momentum of $B$ after the collision must be greater than the value found in part **(i)**. Deduce that $b$ must be greater than the value found in part **(ii)**. [3]

3




Three forces, of magnitudes 8 N, 10 N and 8 N, act at a point $P$ in the directions shown in the diagram. $PQ$ is the bisector of the acute angle between the two forces of magnitude 8 N. Find

**(i)** the components of the resultant of the three forces

**(a)** parallel to $PQ$,

**(b)** perpendicular to $PQ$,

[4]

**(ii)** the magnitude of the resultant of the three forces, [2]

**(iii)** the angle that the resultant of the three forces makes with $PQ$. [2]

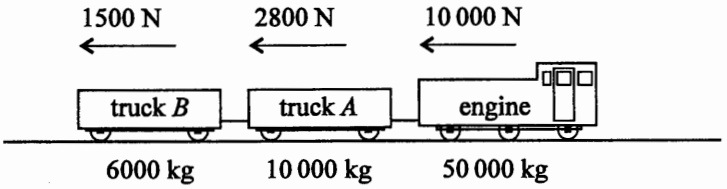
4 A box of mass 200 kg rests in equilibrium on a plane inclined at 32° to the horizontal.

**(i)** Calculate the frictional force acting on the box. [2]

**(ii)** Given that the equilibrium is limiting, calculate the coefficient of friction between the box and the plane. [3]

**(iii)** Calculate the maximum force, acting up the slope, which can be applied to the box without causing the box to slip. [4]

5




A train consists of an engine of mass 50 000 kg coupled to two trucks $A$ and $B$ of masses 10 000 kg and 6000 kg respectively (see diagram). The couplings are light, rigid and horizontal. The train moves along a horizontal track with a constant deceleration. The resistances to motion of the engine, truck $A$ and truck $B$ are 10 000 N, 2800 N and 1500 N respectively. The tension in the coupling between truck $A$ and truck $B$ is zero.

**(i)** By applying Newton's second law to truck $B$, show that the deceleration of the train is 0.25 m s$^{-2}$. [2]

**(ii)** Find the tension in the coupling between the engine and truck $A$. [3]

**(iii)** Determine whether the engine exerts a driving force or a braking force, and find its magnitude. [4]

6 A particle of mass $M$ kg slides down a rough plane which is inclined at 30° to the horizontal. The particle passes through a point $A$ with speed 4 m s$^{-1}$, and 2 s later it passes through a point $B$ with speed 9 m s$^{-1}$.

**(i)** Find the acceleration of the particle. [2]

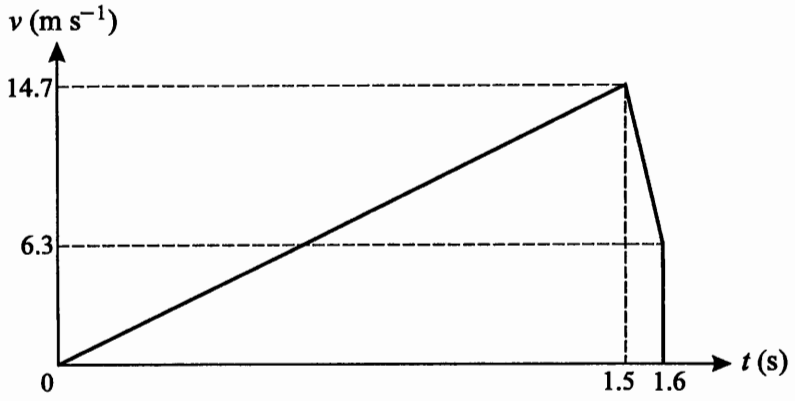
**(ii)** Find the distance $AB$. [2]

**(iii)** Show that the frictional force on the particle is $2.4M$ newtons. [3]

**(iv)** Find the coefficient of friction between the particle and the plane. [3]

**[Question 7 is printed overleaf.]**

7




A stone falls vertically from rest, under gravity, into a drum of oil. The diagram shows the $(t, v)$ graph for the motion of the stone, where $v$ m s$^{-1}$ is the stone's downwards velocity at time $t$ seconds after it starts to fall.

**(i)** Use the information in the diagram to describe briefly the motion of the stone after it reaches the surface of the oil. [2]

**(ii)** Find the height, above the surface of the oil, from which the stone falls. [2]

**(iii)** Find the depth of oil in the drum. [2]

**(iv)** Find the deceleration of the stone when $t = 1.55$. [2]

**(v)** The mass of the stone is 0.08 kg. Find the upward force acting on the stone, due to the oil, when $t = 1.55$. [3]

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| **1** | **(i)** | Velocity is $10t - 1.5t^2$<br>Acceleration is $10 - 3t$ | M1<br>A1<br>B1 ft | 3 | For using $v = \dot{s}(t)$ |
| | **(ii)** | $10 - 3t = 4 \rightarrow t = 2$<br>$v = 10 \times 2 - 1.5 \times 2^2$<br>Velocity is 14 $ms^{-1}$ | M1*<br>M1dep*<br>A1 | 3 | For setting $a(t) = 4$ and attempting to solve for $t$<br>For substituting the value of $t$ found into $v(t)$ |

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| **2** | **(i)** | $3 \times 0.15 - 2 \times 0.12$<br>Magnitude is 0.21 Ns | M1<br>A1 | 2 | For using the idea that the sum of the momentum is $\Sigma mv$ (explicitly, not as part of an equation derived from applying the principle of conservation of momentum)<br>Accept $3 \times 0.15$ and $-2 \times 0.12$ for M1<br>This mark can be scored in (ii) providing the M1 is scored in (i) |
| | **(ii)** | $0.12b$ = ans (i)<br>$b = 1.75$ | M1<br>A1 | 2 | For using the principle of conservation of momentum |
| | | | | | SR (max 1 mark out of 2)<br>$0.12gb = 0.21g \rightarrow b = 1.75$ B1 |
| | **(iii)** | Contribution from $A$ is -ve<br>$0.12b > 0.21 \rightarrow b > 1.75$ | B1<br>M1<br>A1 ft | 3 | Allow even if expressions for momenta include $g$<br>For using $0.12b >$ ans (i) |
| | | | | | SR for the above two marks (max 1 out of 2 marks)<br>$0.12bg > 0.21g \rightarrow b > 1.75$ B1 |

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| **3** | **(i)(a)** | $2 \times 8\cos 40^o - 10\cos 10^o$<br>Component of resultant is 2.41 N | M1<br>A1<br>A1 ft | | For resolving in the direction of $\vec{PQ}$ (or $\vec{QP}$)<br>Allow minus this expression if the direction of resolving is stated or implied to be to the left<br>Allow – 2.41. ft wrong angle $10^o$ |
| | **(b)** | Component of resultant is 1.74 N | B1 ft | 4 | ft wrong angle $10^o$<br>Accept $10\sin 10^o$ providing 1.74 is seen in (ii) and/or (iii). |
| | | | | | SR (max 2 out of 4 marks)<br>(a) $8\cos 40^o$, $-10\cos 10^o$, $8\cos 40^o$ B1<br>(b) $8\sin 40^o$, $10\sin 10^o$, $-8\sin 40^o$ B1<br>NB Q3(i) must be marked either according to the main scheme or according to the SR; it must not be marked according to a part of one and a part of the other. |
| | **(ii)** | $R^2 = 2.41^2 + 1.74^2$<br>Magnitude is 2.97 N | M1<br>A1 ft | 2 | For using $R^2 = X^2 + Y^2$<br>ft wrong answer(s) in (i) |
| | **(iii)** | $\tan\theta = 1.74/2.41$<br>Angle is $35.8^o$ (or $35.9^o$) | M1<br>A1 ft | 2 | For using $\tan\theta =$ (i)($b$)/(i)($a$)<br>ft wrong answer(s) in (i) |

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| **4** | **(i)** | $F = 200\times 9.8\sin 32^o$ | M1 | | For using the idea that the frictional force is equal to the component of weight parallel to the plane |
| | | Force is 1040 N | A1 | 2 | |
| | **(ii)** | $R = 200\times 9.8\cos 32^o$ | B1ft | | ft sin/cos mix |
| | | $\mu$ = 1039/1662 | M1 | | For using $\mu = F/R$ |
| | | | | | Alternatively: for $\mu = \tan 32^o$ M2 |
| | | Coefficient is 0.625 | A1 | 3 | |
| | **(iii)** | | M1 | | For resolving forces parallel to the plane |
| | | $P_{max} = F_{max} + 200\times 9.8\sin 32^o$ | A1ft | | ft sin/cos mix |
| | | $F_{max}$ = 1040 | B1 ft | | |
| | | Maximum force is 2080 N | A1 ft | 4 | ft $2F_{max}$ |

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| **5** | **(i)** | $6000a = (-)1500$ | B1 | | |
| | | Deceleration is 0.25 $ms^{-2}$ | B1 | 2 | or $a$ = - 0.25 (or a = 0.25 if it is clear that the direction of the acceleration is taken to be opposite the direction of motion) |
| | **(ii)** | | M1 | | For applying Newton's second law to truck $A$ or to both trucks combined |
| | | $T - 2800 = 10000(-0.25)$ | A1 | | or $T - 2800 - 1500 =$ $(10000 + 6000)(-0.25)$ |
| | | The tension is 300 N | A1 | 3 | |
| | **(iii)** | | M1 | | For applying Newton's second law to the engine or to the whole system |
| | | Forward force –10000 – ans(ii) = 50000(-0.25) | A1 | | or Forward force – (10000 + 2800 + 1500) = (50000 + 10000 + 6000)(-0.25) |
| | | Braking force | A1 ft | | |
| | | Magnitude 2200 N | A1 | 4 | |

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| **6** | **(i)** | $9 = 4 + 2a$ | M1 | | For using $v = u + at$ |
| | | Acceleration is 2.5 $ms^{-2}$ | A1 | 2 | |
| | **(ii)** | $AB/2 = (4 + 9)/2$ or | M1 | | For using $s/t = (u + v)/2$ or $s = ut + \frac{1}{2}at^2$ |
| | | $AB = 4\times 2 + \frac{1}{2}\, 2.5\times 2^2$ or $81 = 16 + 5(AB)$ | | | or $v^2 = u^2 + 2as$ |
| | | Distance $AB$ is 13 m | A1 ft | 2 | ft for 8 + 2ans(i) or 32.5 ÷ ans(i) |
| | **(iii)** | | M1 | | For applying Newton's second law parallel to the plane (3 terms needed) |
| | | $Mg\sin 30^o - F = Ma$ | A1 | | |
| | | $F = 2.4M$ | A1 | 3 | |
| | **(iv)** | $R = Mg\cos 30^o$ | B1 | | Must contain $M$ unless 'let $M$ = some value' is clearly stated |
| | | $\mu = 2.4/4.9\sqrt{3}$ | M1 | | For applying $\mu = F/R$ |
| | | Coefficient is 0.283 or $\frac{8}{49}\sqrt{3}$ | A1 | 3 | Allow following the use of a specific value of $M$. |

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| **7** | **(i)** | Stone is slowing down<br>Stops or<br>Hits the bottom of the drum | B1<br>B1 2 | |
| | **(ii)** | ½ $14.7 \times 1.5$ or ½ $9.8 \times 1.5^2$<br><br>Height is 11.025 m | M1<br><br>A1 2 | For using the idea that the area of the relevant triangle represents the height or for using $h = ½ gt^2$<br>or 11.0 m |
| | **(iii)** | ½ $\times (14.7 + 6.3) \times 0.1$<br><br>Depth is 1.05 m | M1<br><br>A1 2 | For using the idea that the area of the relevant trapezium represents the depth or for using $s = ½ (u + v)t$ |
| | **(iv)** | $a = (6.3 – 14.7)/0.1$<br><br>Deceleration is 84 $ms^{-2}$ | M1<br><br>A1 2 | For using the idea that the gradient represents the acceleration |
| | **(v)** | <br><br>$F = 0.08 \times (84 + g)$<br>Upward force is 7.504 N | M1<br><br>A1 ft<br>A1 3 | For applying Newton's second law to the stone (3 terms needed)<br>AEF<br>or 7.50 N |
| | | | | SR (to deal with the case of omission of the weight) (max 1 out of 3)<br>Upward force is 6.72 N B1 |